X-616-69-281
PREPRINT

NASA TM X-63610

ANISOTROPIC SOLAR COSMIC RAY PROPAGATION IN AN INHOMOGENEOUS MEDIUM

I SOLAR ENVELOPE

L. F. BURLAGA

JULY 1969

GSFC
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-616-69-281

ANISOTROPIC COSMIC RAY PROPAGATION IN AN INHOMOGENEOUS MEDIUM *

I. The Solar Envelope

Leonard F. Burlaga
Laboratory for Space Sciences
NASA-Goddard Space Flight Center
Greenbelt, Maryland USA

July 1969

*Presented at Eleventh International Conference on Cosmic Rays Budapest, Hungary, 25 August 1969.

Extraterrestrial Physics Branch Preprint Series

ABSTRACT

ANISOTROPIC SOLAR COSMIC RAY PROPAGATION
IN AN INHOMOGENEOUS MEDIUM

A phonomenological, 1-dimensional model of solar cosmic ray propagation is presented which allows one to compute both the anisotropy and the intensity as a function of time during the early phases of certain events. It applies for arbitrarily large anisotropies, ranging from 0 to 1, and is valid for even the earliest times. Thus, it is not subject to the restrictions of earlier models based on the diffusion equation of telegraph equation.

The model assumes that cosmic rays are released instantaneously and effectively interact with isolated scattering centers which are distributed uniformly throughout the solar wind. Each scattering center is assigned a probability $P(x_i)$ that a cosmic ray will be reflected by it; this function ranges from 0 to 1 and can be any function of position. The mathematical formulation of the model consists of a set of difference equations involving $P(x_i)$ and the probabilities f^+ and f^- of finding a particle moving away from or toward the source, respectively, at a given time and position.

The model is used to examine the general effects of a diffusing region close to the sun (the solar envelope) on solar cosmic ray propagation. The model, applied to the >1.1 bv protons of the May 4, 1960, event, gives a good fit to the intensity-time curve for the "sunward" flux and correctly gives a negligible "anti-sun" flux throughout the event. The characteristic dimension and diffusion coefficient of the diffusing region are found to be 27 R⊙ and $D_{max} = 10^{21}$ cm^2/sec,

respectively. It is suggested that the solar envelope may be identified with an extended solar wind heating region and that the cosmic-ray scattering may be due to the hydromagnetic waves which Barnes has postulated as the energy source.

ANISOTROPIC COSMIC RAY PROPAGATION IN AN INHOMOGENEOUS MEDIUM

I. The Solar Envelope

L. F. Burlaga

I. Introduction

The existence of a diffusing region close to the sun has been inferred from recent observations of low energy, solar cosmic ray anisotropies (McCracken et al., 1967 , Lin et al. (1968)) and from an early study of the anisotropy and dispersion effect in a high energy ($\gtrsim$1 Bev) solar cosmic ray event (Lust and Simpson, 1957). Following Lust and Simpson, we shall call this region the "solar envelope". The above authors, also infer the existence of a second diffusing region near the earth and beyond, which is the cause of the trend toward isotropy with increasing time. Following Parker, who was the first to study this region quantitatively, we shall refer to it as the "solar shell." The large anisotropies observed during the rise of certain events imply that relatively little diffusion occurs between the solar envelope and the solar shell.

There are many diffusion madels which attempt to examine various aspects of the propagation, but as yet there is no model which properly accounts for the highly inhomogeneous structure of the solar wind as described above, and which describes <u>both</u> the solar cosmic ray intensity <u>and</u> anisotropy as a function of time. The aim of this paper is to present a simple, phenomenological, 1-dimensional model, valid even in the limit of large anisotropies, which enables one to use the measured anisotropy and intensity profiles of solar cosmic rays to study the inhomogeneous structure of the medium.

The model is presented in Section 2. In Section 3 the model is applied to examine the general effects of the solar envelope on cosmic ray propagation, and it is shown how the model can be used to analyze certain types of cosmic ray events. The unusual event on May 4, 1960, is analyzed in Section 4, where it is shown that the model gives good fits to the intensity-time profiles for both the forward and backward fluxes, and that the size of the solar envelope is $\sim 27R_{\odot}$. A possible relation between the solar envelope and the solar wind heating region is also discussed in Section 4.

For most events, the solar shell must also be considered, because this is the cause of the approach to isotropy. This will be considered in a subsequent publication, where analytical properties of the model and its relation to models based on the Boltzman equation will also be discussed

II. The Model

We consider propagation in a 1-dimensional, semi-infinite medium with a source at x=0. The medium is represented by an infinite number of point scattering centers which are equally spaced and extend from the source to infinity. The scattering properties of the medium are described by two parameters: λ, the constant separation between the scattering centers, and the "effectiveness" of the scattering centers, P_L, which is the probability that a particle will be reflected when it encounters a scattering center at x_L. The properties of the cosmic rays are represented by three parameters: a) their speed v, which is assumed to be constant, b) $f^+_{L,T}$, the probability that a particle is moving outward (i.e. away from the source) after a collision at L-1, T-1 and is just approaching point L at time T and c) $f^-_{L,T}$, the probability that a particle is moving inward (i.e., toward the source) after a collision at L+1, T-1 and is just approaching point L at time T. Thus, $f^+_{L,T}$ is proportional to the flux of particles moving away from the source (the forward flux) and $f^-_{L,T}$ is proportional to the flux of particles moving toward the source (the backward flux).

One can write the following recurrence equations for f^+ and f^-:

$$
\begin{aligned}
f^+_{L+1,T+1} &= P_L\, f^-_{L,T} + (1-P_L)\, f^+_{L,T} \\
f^-_{L-1,T+1} &= (1-P_L)\, f^-_{L,T} + P_L\, f^+_{L,T}
\end{aligned}
\qquad (1)
$$

The first of these equations states that the probability that a particle is moving outward toward L+1 at time T+1 is the sum of the probability that a particle moves inward toward L at T and is reflected at that point, plus the probability that a particles moves outward toward L at T and is not reflected at that point. The second equation has a similar meaning. These equations form the basis for the model. Given P_L, a source function, and the boundary conditions one can use (1) to calculate $f^+_{L,T}$ and $f^-_{L,T}$ at any L, T.

For the source function, we choose

$$f^+_{o,o} = 1; \quad f^-_{o,o} = 0, \tag{2}$$

corresponding to an instantaneous point source. For the boundary condition, we choose

$$f^+_{1,T} = f^-_{o,T-1}, \tag{3}$$

which implies a reflecting boundary at $x_L=0$

III. Application to the Solar Envelope

Ideally, one would like to use the model to deduce $P(x)$. Of course, this cannot be done. However, one can proceed inductively and use the model to test certain hypotheses concerning $P(x)$. This section follows the second approach and examines the effects of a diffusing region close to the sun (the solar envelope) on solar cosmic ray propagation.

Let us assume that

$$P(L) = .5e^{-\left(\frac{L}{L_D}\right)^2} \qquad (4)$$

The choice of a gaussian is somewhat arbitrary, but the results of interest do not depend sensitively on the exact form of $P(L)$. The coefficient .5 was chosen because in the limit $L_D \rightarrow \infty$ the model reduces to the 1-dimensional random walk in a homogeneous medium, if $P = .5$. The essential effects of the solar envelope on cosmic rays are determined by the number of scattering centers in the envelope, which is measured by L_D.

To illustrate the properties of the model and the effects of the envelope, we have calculated the sunward and antisunward "fluxes", f^+ and f^-, for various values of L_D, assuming that there are L_o scattering centers between the observer and the source. The results fall into three classes, corresponding to $L_D \ll L_o$, $L_D \sim L_o$, and $L_D \gtrsim L_o$.

When $L_D \ll L_o$, f^- is found to be negligible, as one would expect, since there is a negligible probability of backscattering beyond the observer's position. The characteristics of f^+ are illustrated in Figure 1a. When $L_D \lesssim 2$, the maximum intensity occurs at the onset, T_o, and $f^+(T)$ decreases exponentially at all times. When L_D=6, the envelope begins to act like a diffusing region; the particle intensity rises to a maximum at Tm, shortly after onset and then decays exponentially as result of the loss of particles from the 'diffusing' region. As L_D increases to 10, the maximum shifts to later times and the decay, though still exponential, is less rapid as a result of the greater effectiveness of the diffusing region in delaying the escape of particles. With the observer at L_o=50, the values L_D = 2,6, and 10 correspond to a solar envelope with dimensions 8.4 $R_\odot$, 25.2 $R_\odot$, and 42 $R_\odot$, respectively.

When $L_D \sim L_o$, the scattering centers beyond L_o are effective, and an appreciable anti sun flux, f^- is observed. This is illustrated in Figure 1b for L_D=30 and L_D=40. The rise to maximum and exponential decay are seen in both f^+ and f^-, and the trend toward increasing T_{max} and T_D with increasing L_o continues. The anisotropy approaches a limiting value which decreases as L_D increases. This is 28% for L_D= 40 and 68% for L_D=30.

When $L_D \gg L_o$, the propagation tends to be like 1-dimensional diffusion, as one would expect, so this case will not be discussed further.

Having illustrated the general effects of a solar envelope of various dimensions on cosmic ray propagation, we now show how certain cosmic ray observations can be used to <u>infer</u> the size R and diffusion coefficient D $\equiv \lambda v/2$, of the solar envelope, on the assumption that P(L) has the general form of (4) .

Since

$$R = \frac{L_D}{L_o} \times 210 \qquad \text{(in units of } R_\odot) \qquad (5)$$

and

$$D\left(\frac{cm^2}{sec}\right) = \frac{1.5 \times 10^{13} v \text{ (cm/sec)}}{2\, L_o} \qquad (6)$$

the problem can be stated, "Given $f^+(t)$ and $f^-(t)$, and assuming $P(x)$ is given by (4), find L_o and L_D."

From numerical calculations with L_o ranging from 50 to 500 and L_D ranging from 5 to 50, the following empirical relation was found

$$L_D \simeq 100 \frac{(T_m / T_o - 1)^2}{(T_D / T_o)^2} \; ; \; L_D < L_o/4. \qquad (7)$$

The error in this relation is less than 10%. When $f^+(t)$ has the form shown in Figure 1, it can be characterized by three parameters: 1) the onset time, $t_o = x_o/v$, where x_o is the position of the observer, 2) the time of maximum, t_m, and 3) the decay time t_D. Since $t = T\tau$, where $\tau = \lambda/v$, we have $T_m/T_o = t_m/t_o$ and $T_D/T_o = t_D/t_o$. Thus, L_D can be determined very easily from the observed f^+ using (7).

Similarly, it was found that there is a simple relation between T_D and L_D, which is given in Figure 2. This is independent of L_o when $L_D \ll L_o$, indicating that the observer sees the same decay profile at any position beyond the "diffusing" region and that the decay constant is determined only by the number of effective ($P \sim .5$) scattering centers in the envelope. Thus, given L_D from (7), one obtains T_D from Figure 2. L_o can then be determined from the relation

$$L_o \doteq T_o = t_o\, T_D/t_D \qquad (8)$$

Thus L_o and L_D are determined by (7) and (8) if t_o, t_m and t_D are known, q.e.d.. R and D are then given by (5) and (6).

IV. The May 4, 1960 Event

We shall n show how the model can be applied to give the characteristics of the envelope and the intensity-time profiles of the forward (looking toward the source) and backward (looking away from the source) fluxes for the >1 bv protons in the May 4, 1960, event.

The classical May 4 event presents a special challenge to propagation theories, for it was of unusually short duration, the anisotropy was always essentially 100%, and it defies any attempt to describe it by the common diffusion models. On the other hand, it was clearly associated with an optical flare and a cm radio burst at 1025 UT, so that the injection time can be determined (assuming instantaneous injection); and good measurements of the intensity-time profiles for several viewing directions are available, so that $f^+(t)$ and $f^-(t)$ are well-known. Moreover, the flare occurred at 80°W, which is only $\sim 10^\circ$ from the theoretical position of the base of the interplanetary magnetic field line that passes through the earth (Burlaga, 1967), so the use of a 1-dimensional model is justified if perpendicular diffusion can be neglected near the sun as suggested by Axford (1965), Burlaga (1967) and others.

The data which we use for this event are taken from McCracken (1962), Figure 2.2. For the forward flux, we use the data from the Fort Churchill neutron monitor which was viewing at an angle $\delta \sim 40^\circ$ from the streaming direction. These data are shown by the dots in Figure 3. The backward flux is inferred to be zero for all t, because Mirny, which viewed at $\delta \sim 120^\circ$ saw no enhancement at any time. Since

f^- was always near the background level, and since f^+ was characterized by a rise to maximum followed by an exponential decay, it is likely that most of the scattering occurred near the sun, so that the model of the preceding section should be applicable for this particular event.

From Figure 3 we find that t_o = 13.3 min, t_m = 22.5 min. and t_D = 20.1 m in. Thus, (7) gives L_D = 20.9, Figure 2 gives T_D = 241, and (8) gives L_o = 160. The collision probability P(x) (equation (4)) is now determined, and the model can be used to calculate f^+ and f^-. The results for $f^+(t)$ are shown by the curve in Figure 3, which is normalized to the maximum intensity. It can be seen that the theory gives an excellent fit to the observations. The corresponding results for $f^-(t)$ (which, recall, is on an equal footing with $f^+(t)$ in our model) show that the maximum is $<10^{-24}$ in the units of Figure 3. Thus the predicted backward flux is essentially zero, in agreement with observations. We conclude that the model provides a satisfactory description of the observed forward and backward fluxes of the >1.1 bv protons in the May 4, 1960 event.

Now let us examine what the model tells about the solar envelope on May 4, 1960. Since L_o = 160, implying 160 scattering centers between the sun and the earth, the separation of the scattering centers was λ = .00625 AU. This is approximately equal to the mean free path near the sun. Using (6) with $v=1.9\times10^{10}$cm/sec., we find that the maximum value of the diffusion coefficient in the solar envelope was $D\sim9\times10^{20}cm^2/sec$. Similarly, using L_D = 20.9 we find from (5) that the size of the diffusing region was $R=27\ R_\odot$.

Note that the decay time t_D = 20 min., is approximately the time required to diffuse out of the shell, $R^2/(4D)$ = 17 min.

Our diffusion coefficient may be compared with the estimate of Shishov (1966), $D \sim 5 \times 10^{20} cm^2/sec$, based on the assumption that $R \sim 10 R_\odot$ on May 4, 1960. His estimate is inferred from an analytical model for propagation in a spherically symmetrical, homogeneous medium from a source of the form exp(-t/16 min). While our model confirms Shishov's intuition, it differs fundamentally from his approach and provides more precise results. Our model might also be compared with those of Axford (1965) and Fisk and Axford (1969). They presented equations for f^+ and f^-, but these are for a homogeneous medium. The solutions cannot describe the May 4, event, We also note that Reid (1964) and Axford (1965) presented models, postulating a very thin ($R \ll 1 R_\odot$) envelope to give diffusion around the sun.

Finally, let us consider the origin of the diffusing region. Burlaga and Ogilvie (1969) presented evidence in support of the hypothesis that there is an extended heat source near the sun which accelerates and heats the solar wind. Hartle and Barnes (1969) have shown the relation between bulk speed and temperature of the solar wind, found by Burlaga and Ogilvie, can be obtained by assuming that protons are heated out to $\sim 20 R_\odot$ at quiet times. Note that the size of the heating region is approximately the size of the solar envelope inferred from our model. Thus, we offer the hypothesis that the heating region may be identified with the solar envelope. This is supported by the suggestion of Barnes (1969) that the heating is produced by damping of hydromagnetic waves, for these waves could act as cosmic ray scattering centers. This hypothesis raises the interesting possibility of using cosmic ray observations and our model to study the solar wind heating region. Since Hartle and Barnes find that a larger heating region is required to give higher solar wind temperatures, we would expect that the size of the envelope inferred from cosmic ray observations would be positively correlated with the solar wind temperature.

ACKNOWLEDGEMENTS

The author thanks Drs. K. W. Ogilvie, N. F. Ness and E. Roelof for their comments on the manuscript. The programming was done by Mr. Thurston Carlton.

REFERENCES

Axford, W. I., 1965, Planet. Space Sci., 13, 1301.

Barnes, Aaron, 1969, Ap. J., 155, 311.

Burlaga, L. F., 1967, J. Geophys. Res., 72, 4449.

Burlaga, L. F., and Ogilvie, K. W., 1969, NASA-GSFC X-616-69-142.

Fisk, L. A. and Axford, W. I., 1969, Solar Physics (to appear)

Hartle, R. E. and Barnes, Aaron (to be published)

Lin, R. P., Kahler, S. W., and E. C. Roelof, (to be published)

Lust, R. and Simpson, J. A., 1957, Phys. Rev., 108, 1563.

McCracken, K. G., 1962, J. Geophys. Res., 447.

McCracken, K. G., Rao, U. R., and Bukata, R. P., 1967, J. Geophys. Res., 72, 4293.

Reid, G. C., 1964, J. Geophys. Res., 69, 2659.

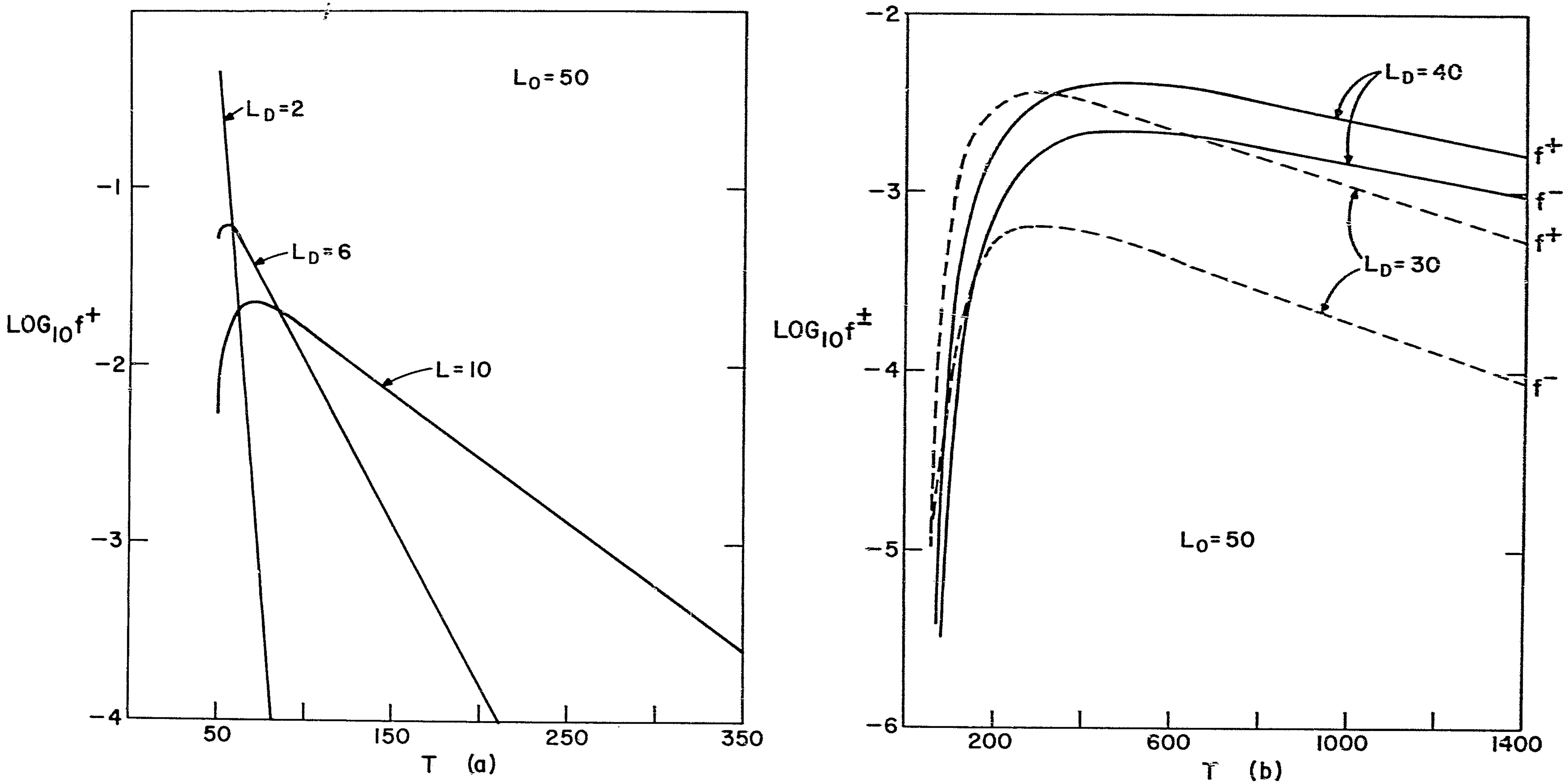
Shishov, V. I., 1966, Geomagn. i aeronomiya, 4, 223.

FIGURE CAPTIONS

Figure 1. Variation of f^+ and f^- with T for a particle released at T=0 into a medium with $F(L) = e^{-(L/L_D)^2}$. The observer is at L_o=50. For $L_D \leq 10$ (panel (a)), f^- is negligible. As L_D approaches L_o, f^- approaches f^+ (panel (b)).

Figure 2. This shows how T_D is related to L_D. The relation is independent of L_o, when $L_D \leq L_o/4$.

Figure 3. The points are observations of >1.1 Bv protons seen at Fort Churchill on May 4, 1960. The curve shows f^+ obtained from the model used to generate the curves in Figure 1; no free parameters were used to get this fit.



LOG10f+
L0=50
LD=2
LD=6
L=10
-1
-2
-3
-4
50
150
250
350
T (a)
LOG10f±
LD=40
LD=30
f+
f−
f+
f−
L0=50
-2
-3
-4
-5
-6
200
600
1000
1400
T (b)

Figure 1

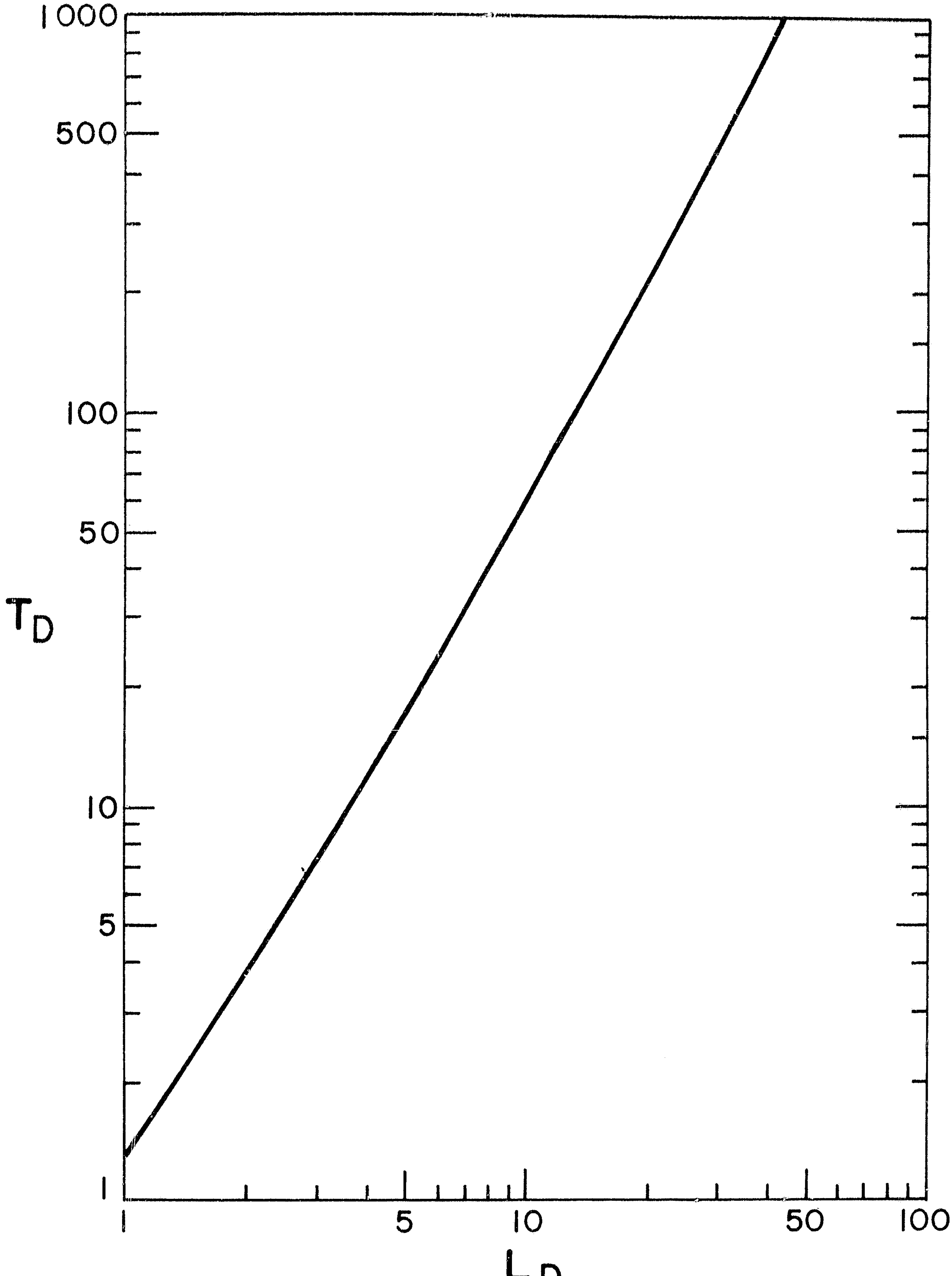
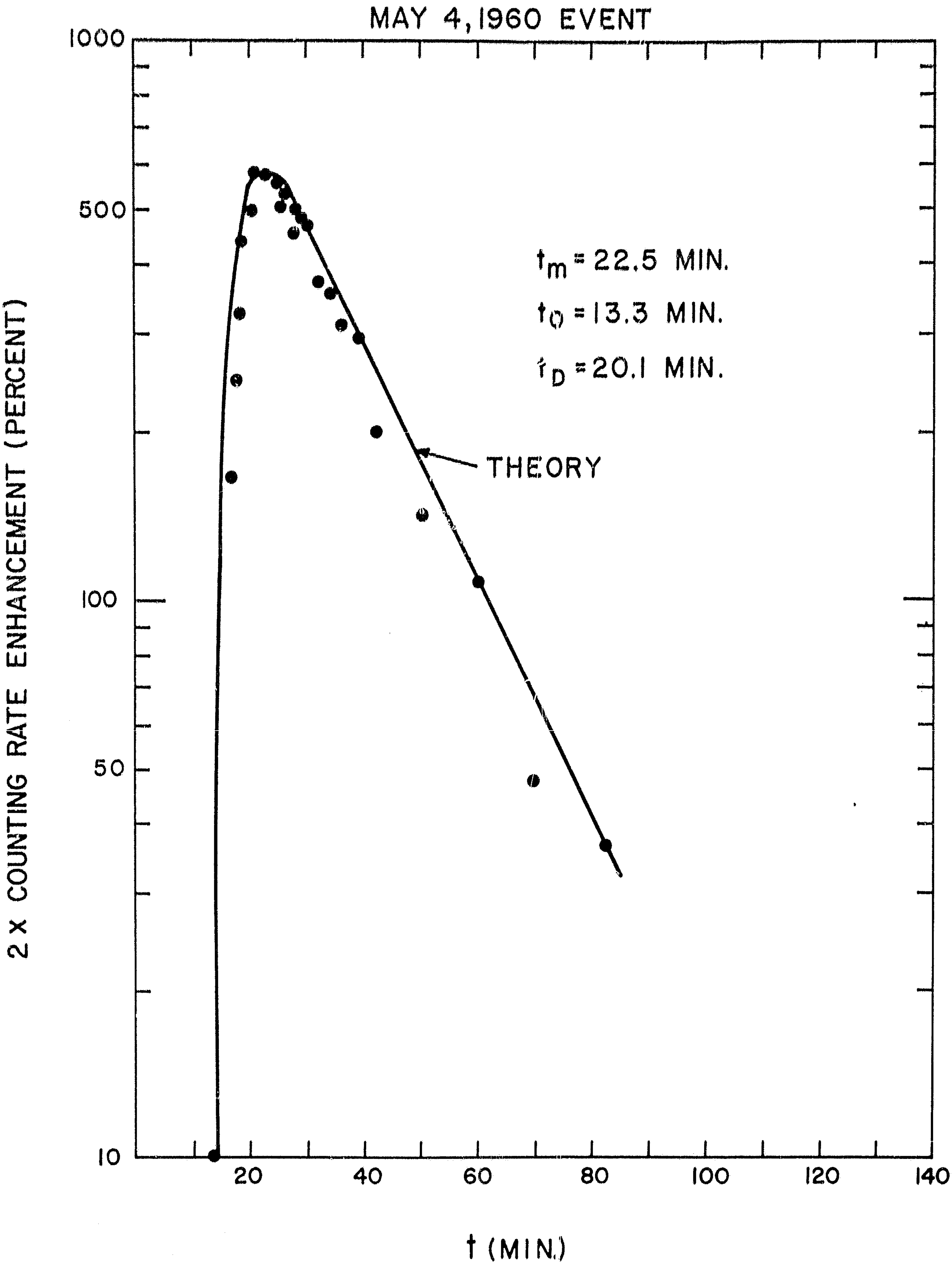


Figure 2



MAY 4, 1960 EVENT
1000
500
100
50
10
2 x COUNTING RATE ENHANCEMENT (PERCENT)
t_m = 22.5 MIN.
t_0 = 13.3 MIN.
t_D = 20.1 MIN.
THEORY
20
40
60
80
100
120
140
t (MIN.)

Figure 3